



V2V EDTECH LLP

Online Coaching at an Affordable Price.

OUR SERVICES:

- Diploma in All Branches, All Subjects
- Degree in All Branches, All Subjects
- BSCIT / CS
- Professional Courses



+91 93260 50669



v2vedtech.com



V2V EdTech LLP



v2vedtech



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Important Instructions to Examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Attempt any <u>FIVE</u> of the following:	10
	a)	State whether the function $f(x) = \frac{e^x + e^{-x}}{2}$ is odd or even.	02
	Ans	$f(x) = \frac{e^x + e^{-x}}{2}$ $\therefore f(-x) = \frac{e^{-x} + e^{-(-x)}}{2}$ $= \frac{e^{-x} + e^x}{2}$ $= f(x)$ <p>\therefore function is even.</p> <p>-----</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
	b)	If $f(x) = \frac{x^2 + 1}{x^3 - 1}$ find $f\left(\frac{1}{2}\right)$	02
	Ans	$f(x) = \frac{x^2 + 1}{x^3 - 1}$ $\therefore f\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^2 + 1}{\left(\frac{1}{2}\right)^3 - 1}$ $= \frac{-10}{7} \quad \text{OR} \quad -1.429$ <p>-----</p>	<p>1</p> <p>1</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$, if $y = (x^2 + 1)^5$	02
	Ans	$y = (x^2 + 1)^5$ $\therefore \frac{dy}{dx} = 5(x^2 + 1)^4 \cdot \frac{d}{dx}(x^2 + 1)$ $= 5(x^2 + 1)^4 \cdot (2x)$ $= 10x(x^2 + 1)^4$	1 1
	d)	Evaluate $\int (\tan x + \cot x)^2 dx$	02
	Ans	$\int (\tan x + \cot x)^2 dx$ $= \int (\tan^2 x + 2 \tan x \cot x + \cot^2 x) dx$ $= \int (\tan^2 x + 2 + \cot^2 x) dx$ $= \int [(\sec^2 x - 1) + 2 + (\operatorname{cosec}^2 x - 1)] dx$ $= \int (\sec^2 x + \operatorname{cosec}^2 x) dx$ $= \tan x - \cot x + c$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
e)	Evaluate $\int \log x dx$	02	
Ans	$\int \log x dx = \int \log x \cdot 1 dx$ $= \log x \int 1 dx - \int \left(\int 1 dx \frac{d}{dx} \log x \right) dx$ $= \log x(x) - \int x \frac{1}{x} dx$ $= x \log x - \int 1 dx$ $= x \log x - x + c$ $= x(\log x - 1) + c$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	
f)	Find the area between the lines $y = 3x$, x -axis and the ordinates $x = 1$ and $x = 5$	02	
Ans	Area $A = \int_a^b y dx$ $= \int_1^5 3x dx$	$\frac{1}{2}$	



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme												
1.	f)	$= 3 \int_1^5 x dx$ $= 3 \left[\frac{x^2}{2} \right]_1^5$ $= 3 \left[\frac{5^2}{2} - \frac{1^2}{2} \right]$ $= 36$	<p>1/2</p> <p>1/2</p> <p>1/2</p>												
	g)	<p>Show that there exist a root of the equation $x^2 - 2x - 1 = 0$ in $(-1, 0)$ and find approximate value of the root by using Bisection method. (Use two iterations)</p>	02												
	Ans	$x^2 - 2x - 1 = 0$ $f(x) = x^2 - 2x - 1$ $f(-1) = 2$ $f(0) = -1$ root is in $(-1, 0)$ $\therefore x_1 = \frac{-1+0}{2} = -0.5$ $\therefore f(-0.5) = 0.25$ \therefore root is in $(-0.5, 0)$ $\therefore x_2 = \frac{-0.5+0}{2} = -0.25$	<p>1/2</p> <p>1/2</p> <p>1/2</p>												
		OR													
		$x^2 - 2x - 1 = 0$ $f(x) = x^2 - 2x - 1$ $f(-1) = 2$ $f(0) = -1$ root is in $(-1, 0)$													
		<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>a</th> <th>b</th> <th>$x = \frac{a+b}{2}$</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>-1</td> <td>0</td> <td>-0.5</td> <td>0.25</td> </tr> <tr> <td>-0.5</td> <td>0</td> <td>-0.25</td> <td>----</td> </tr> </tbody> </table>	a	b	$x = \frac{a+b}{2}$	f(x)	-1	0	-0.5	0.25	-0.5	0	-0.25	----	<p>1</p> <p>1</p>
a	b	$x = \frac{a+b}{2}$	f(x)												
-1	0	-0.5	0.25												
-0.5	0	-0.25	----												



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		Attempt any <u>THREE</u> of the following:	12
	a)	Find $\frac{dy}{dx}$ if $\cos(x^2 + y^2) = \log(xy)$	04
	Ans	$\cos(x^2 + y^2) = \log(xy)$ $\therefore -\sin(x^2 + y^2)\left(2x + 2y \frac{dy}{dx}\right) = \frac{1}{xy}\left(x \frac{dy}{dx} + y\right)$ $\therefore -2x \sin(x^2 + y^2) - 2y \sin(x^2 + y^2) \frac{dy}{dx} = \frac{1}{y} \frac{dy}{dx} + \frac{1}{x}$ $\therefore \frac{dy}{dx}\left(-2y \sin(x^2 + y^2) - \frac{1}{y}\right) = \frac{1}{x} + 2x \sin(x^2 + y^2)$ $\therefore \frac{dy}{dx} = \frac{\frac{1}{x} + 2x \sin(x^2 + y^2)}{-2y \sin(x^2 + y^2) - \frac{1}{y}}$	2 1 1
	b)	If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$	04
	Ans	$x = a \cos^3 \theta$ $y = a \sin^3 \theta$ $\therefore \frac{dx}{d\theta} = -3a \cos^2 \theta \sin \theta$ $\frac{dy}{d\theta} = 3a \sin^2 \theta (\cos \theta)$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\frac{dy}{dx} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta}$ $\frac{dy}{dx} = -\tan \theta$ at $\theta = \frac{\pi}{4}$ $\frac{dy}{dx} = -\tan \frac{\pi}{4}$ $\frac{dy}{dx} = -1$	1 1 1



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	Find the maximum and minimum value of $2x^3 - 3x^2 - 36x + 10$	04
	Ans	Let $y = 2x^3 - 3x^2 - 36x + 10$ $\therefore \frac{dy}{dx} = 6x^2 - 6x - 36$ $\therefore \frac{d^2y}{dx^2} = 12x - 6$ Consider $\frac{dy}{dx} = 0$ $6x^2 - 6x - 36 = 0$ $x^2 - x - 6 = 0$ $\therefore x = -2, x = 3$ at $x = -2$ $\frac{d^2y}{dx^2} = 12(-2) - 6 = -30 < 0$ $\therefore y$ is maximum at $x = -2$ $y_{\max} = 2(-2)^3 - 3(-2)^2 - 36(-2) + 10$ $= 54$ at $x = 3$ $\frac{d^2y}{dx^2} = 12(3) - 6 = 30 > 0$ $\therefore y$ is minimum at $x = 3$ $y_{\min} = 2(3)^3 - 3(3)^2 - 36(3) + 10$ $= -71$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	d)	A beam is bent in the form of the curve $y = 2 \sin x - \sin 2x$ Find the radius of curvature of the beam at the point $x = \frac{\pi}{2}$	04
	Ans	$y = 2 \sin x - \sin 2x$ $\therefore \frac{dy}{dx} = 2 \cos x - 2 \cos 2x$ $\therefore \frac{d^2y}{dx^2} = -2 \sin x + 4 \sin 2x$ \therefore at $x = \frac{\pi}{2}$ $\frac{dy}{dx} = 2 \cos\left(\frac{\pi}{2}\right) - 2 \cos 2\left(\frac{\pi}{2}\right) = 2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\frac{d^2y}{dx^2} = -2\sin\left(\frac{\pi}{2}\right) + 4\sin 2\left(\frac{\pi}{2}\right) = -2$ $\therefore \text{Radius of curvature is } \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ $\therefore \rho = \frac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2}$ $\therefore \rho = -5.590 \text{ or } 5.590$	<p>½</p> <p>1</p> <p>1</p>
3.		<p>Attempt any THREE of the following:</p>	12
	a)	<p>Find the points on the curve $y = x^3 + 3x^2 - 9x + 7$ at which tangents drawn are parallel to x-axis.</p>	04
	Ans	$y = x^3 + 3x^2 - 9x + 7$ $\frac{dy}{dx} = 3x^2 + 6x - 9$ <p>\therefore tangent is parallel to x-axis</p> <p>\therefore slope of tangent = slope of x-axis</p> $\therefore \frac{dy}{dx} = 0$ $\therefore 3x^2 + 6x - 9 = 0$ $\therefore x = 1 \ ; \ x = -3$ $\therefore y = 2 \ ; \ y = 34$ <p>\therefore points are $(1, 2)$ and $(-3, 34)$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
	b)	<p>Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$</p>	04
	Ans	<p>Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$</p> <p>Put $x = \tan \theta \Rightarrow \tan^{-1} x = \theta$</p> $u = \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$ $u = \tan^{-1}(\tan 2\theta)$ $u = 2\theta$ $u = 2 \tan^{-1} x$	<p>½</p> <p>½</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\frac{du}{dx} = \frac{2}{1+x^2}$	½
		$v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$	
		$v = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$	½
		$v = \sin^{-1}(\sin 2\theta)$	½
		$v = 2\theta$	
		$v = 2 \tan^{-1} x$	
		$\frac{dv}{dx} = \frac{2}{1+x^2}$	½
		$\therefore \frac{du}{dv} = \frac{dx}{dv} = \frac{1+x^2}{2}$	½
		$\frac{dx}{1+x^2}$	
		$\therefore \frac{du}{dv} = 1$	½
		<i>OR</i>	
		Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$	
		$\therefore \frac{du}{dx} = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2} \times \left[\frac{(1-x^2)2 - 2x(-2x)}{(1-x^2)^2} \right]$	1
		$\therefore \frac{du}{dx} = \frac{(1-x^2)^2}{(1-x^2)^2 + 4x^2} \left[\frac{2+2x^2}{(1-x^2)^2} \right]$	
		$\therefore \frac{du}{dx} = \frac{2+2x^2}{(1-x^2)^2 + 4x^2}$	
		$\therefore \frac{du}{dx} = \frac{2(1+x^2)}{(1-x^2)^2 + 4x^2}$	
		$\therefore \frac{du}{dx} = \frac{2(1+x^2)}{(1+x^2)^2} = \frac{2}{1+x^2}$	½
		$\therefore v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$	



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: **22224**

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\therefore \frac{dv}{dx} = \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \times \left[\frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2} \right]$ $\therefore \frac{dv}{dx} = \frac{(1+x^2)}{\sqrt{(1+x^2)^2 - 4x^2}} \left[\frac{2-2x^2}{(1+x^2)^2} \right]$ $\therefore \frac{dv}{dx} = \frac{(2-2x^2)}{(1+x^2)\sqrt{(1+x^2)^2 - 4x^2}}$ $\therefore \frac{dv}{dx} = \frac{2(1-x^2)}{(1+x^2)(1-x^2)}$ $\therefore \frac{dv}{dx} = \frac{2}{(1+x^2)}$ $\therefore \frac{du}{dv} = \frac{dv}{du} = \frac{2}{(1+x^2)}$ $\therefore \frac{du}{dv} = 1$	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
	c)	<p>Find $\frac{dy}{dx}$ if $y = (\log x)^x + x^{\cos^{-1}x}$</p> <p>Ans Let $u = (\log x)^x$</p> <p>$\log u = x \log (\log x)$</p> <p>$\frac{1}{u} \frac{du}{dx} = x \frac{1}{\log x} \frac{1}{x} + \log (\log x)$</p> <p>$\therefore \frac{du}{dx} = u \left(\frac{1}{\log x} + \log (\log x) \right)$</p> <p>$\therefore \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log (\log x) \right]$</p> <p>Let $v = x^{\cos^{-1}x}$</p> <p>$\log v = \cos^{-1}x \log x$</p>	<p>04</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$\frac{1}{v} \frac{dv}{dx} = \cos^{-1} x \left(\frac{1}{x} \right) + \log x \left(\frac{-1}{\sqrt{1-x^2}} \right)$	1/2
		$\therefore \frac{dv}{dx} = x^{\cos^{-1} x} \left[(\cos^{-1} x) \left(\frac{1}{x} \right) - \log x \left(\frac{1}{\sqrt{1-x^2}} \right) \right]$	1/2
		$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\therefore \frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + x^{\cos^{-1} x} \left[(\cos^{-1} x) \left(\frac{1}{x} \right) - \log x \left(\frac{1}{\sqrt{1-x^2}} \right) \right]$	1/2
	d)	Evaluate: $\int \frac{\sec x \cos ecx}{\log \tan x} dx$	04
	Ans	$\int \frac{\sec x \cos ecx}{\log \tan x} dx$ Put $\log \tan x = t$ $\therefore \frac{1}{\tan x} \sec^2 x dx = dt$ $\therefore \frac{\cos x}{\sin x} \frac{1}{\cos^2 x} dx = dt$ $\therefore \sec x \cos ecx dx = dt$ $= \int \frac{1}{t} dt$ $= \log t + c$ $= \log(\log(\tan x)) + c$	1 1 1/2 1 1/2
4.		Attempt any <u>THREE</u> of the following:	12
	a)	Evaluate : $\int \frac{1}{2x^2 + 3x + 1} dx$	04
	Ans	$\int \frac{1}{2x^2 + 3x + 1} dx$ $= \frac{1}{2} \int \frac{1}{x^2 + \frac{3}{2}x + \frac{1}{2}} dx$	1/2



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	<p>Third term = $\left(\frac{1}{2} \times \frac{3}{2}\right)^2 = \frac{9}{16}$</p> $= \frac{1}{2} \int \frac{1}{x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16} + \frac{1}{2}} dx$ $= \frac{1}{2} \int \frac{1}{\left(x + \frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2} dx$ $= \frac{1}{2} \left[\frac{1}{2\left(\frac{1}{4}\right)} \log \left(\frac{x + \frac{3}{4} - \frac{1}{4}}{x + \frac{3}{4} + \frac{1}{4}} \right) \right] + c$ $= \log \left(\frac{2x+1}{2x+2} \right) + c$ <p style="text-align: center;"><i>OR</i></p> $\int \frac{1}{2x^2 + 3x + 1} dx = \int \frac{1}{(2x+1)(x+1)} dx$ <p>Let $\frac{1}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$</p> $1 = A(x+1) + B(2x+1)$ <p>Put $x = \frac{-1}{2}$</p> $\therefore A = 2$ <p>Put $x = -1$</p> $\therefore B = -1$ $\frac{1}{(2x+1)(x+1)} = \frac{2}{2x+1} + \frac{-1}{x+1}$ $\int \frac{1}{(2x+1)(x+1)} dx = \int \left(\frac{2}{2x+1} + \frac{-1}{x+1} \right) dx$ $= \frac{2 \log(2x+1)}{2} - \log(x+1) + c$ $= \log(2x+1) - \log(x+1) + c$ <p style="text-align: center;"><i>OR</i></p>	<p>1</p> <p>1</p> <p>1½</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>1+1</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	$\int \frac{1}{2x^2 + 3x + 1} dx$ $\text{Third term} = \frac{(M.T.)^2}{4(F.T.)} = \frac{9}{8}$ $= \int \frac{1}{2x^2 + 3x + \frac{9}{8} - \frac{9}{8} + 1} dx$ $= \int \frac{1}{\left(\sqrt{2x} + \frac{3}{\sqrt{8}}\right)^2 - \frac{1}{8}} dx$ $= \int \frac{1}{\left(\sqrt{2x} + \frac{3}{\sqrt{8}}\right)^2 - \left(\frac{1}{\sqrt{8}}\right)^2} dx$ $= \frac{1}{\sqrt{2}} \left[\frac{1}{2\left(\frac{1}{\sqrt{8}}\right)} \log \left(\frac{\sqrt{2x} + \frac{3}{\sqrt{8}} - \frac{1}{\sqrt{8}}}{\sqrt{2x} + \frac{3}{\sqrt{8}} + \frac{1}{\sqrt{8}}} \right) \right] + c$ $= \log \left(\frac{2x+1}{2x+2} \right) + c$	<p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1</p>
	b)	<p>Evaluate : $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$</p> <p>Ans $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$</p> $= \int \frac{dx / \cos^2 x}{\frac{a^2 \sin^2 x + b^2 \cos^2 x}{\cos^2 x}}$ $= \int \frac{\sec^2 x dx}{a^2 \tan^2 x + b^2}$ <div style="display: flex; align-items: center; margin-left: 20px;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-left: 10px;"> $Put \tan x = t$ $\therefore \sec^2 x dx = dt$ </div> </div> $= \int \frac{dt}{a^2 t^2 + b^2}$ $= \int \frac{dt}{a^2 \left(t^2 + \frac{b^2}{a^2} \right)}$	<p>04</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: **22224**

Q. No.	Sub Q. N.	Answer	Marking Scheme	
4.	b)	$= \frac{1}{a^2} \int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2} = \frac{1}{a^2} \cdot \frac{1}{\frac{b}{a}} \tan^{-1} \left(\frac{t}{\frac{b}{a}} \right) + c$	1	
		$= \frac{1}{ab} \tan^{-1} \left(\frac{a \tan x}{b} \right) + c$	1/2	
		<i>OR</i>		
		$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$	1/2	
		$= \int \frac{dx / \cos^2 x}{\frac{a^2 \sin^2 x + b^2 \cos^2 x}{\cos^2 x}}$		
		$= \int \frac{\sec^2 x dx}{a^2 \tan^2 x + b^2}$	1/2	
			<i>Put tan x = t</i> $\therefore \sec^2 x dx = dt$	
			$= \int \frac{dt}{a^2 t^2 + b^2}$	1
			$= \frac{1}{b} \tan^{-1} \left(\frac{at}{b} \right) \frac{1}{a} + c$	1/2
			$= \frac{1}{ab} \tan^{-1} \left(\frac{a \tan x}{b} \right) + c$	1/2
		c)	Evaluate : $\int x \operatorname{cosec}^{-1} x dx$	04
	Ans		$\int x \operatorname{cosec}^{-1} x dx$	
		$= \operatorname{cosec}^{-1} x \int x dx - \int \left(\int x dx \frac{d}{dx} \operatorname{cosec}^{-1} x \right) dx$	1/2	
		$= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \left(\frac{-1}{x\sqrt{x^2-1}} \right) \cdot dx$	1	
		$= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{2} \int \frac{x}{\sqrt{x^2-1}} \cdot dx$		
		$= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{4} \int \frac{2x}{\sqrt{x^2-1}} \cdot dx$	1	
		$= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{4} (2\sqrt{x^2-1}) + c$	1	
		$= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{2} (\sqrt{x^2-1}) + c$	1/2	



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	Evaluate : $\int \frac{1}{x(2-\log x)(2\log x-1)} dx$	04
	Ans	$\int \frac{1}{x(2-\log x)(2\log x-1)} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $Put \log x = t$ $\therefore \frac{1}{x} dx = dt$ </div> $\int \frac{1}{(2-t)(2t-1)} dt$ $\frac{1}{(2-t)(2t-1)} = \frac{A}{2-t} + \frac{B}{2t-1}$ $1 = A(2t-1) + B(2-t)$ $\therefore Put \ t = 2 \ , \ A = \frac{1}{3}$ $Put \ t = \frac{1}{2} \ , \ B = \frac{2}{3}$ $\therefore \frac{1}{(2-t)(2t-1)} = \frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1}$ $\int \frac{1}{(2-t)(2t-1)} dt = \int \left(\frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1} \right) dt$ $= -\frac{1}{3} \log [2-t] + \frac{2}{6} \log [2t-1] + c$ $= -\frac{1}{3} \log [2-\log x] + \frac{1}{3} \log [2\log x-1] + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>
		<p style="text-align: center;">OR</p> $\int \frac{1}{x(2-\log x)(2\log x-1)} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $Put \log x = t$ $\therefore \frac{1}{x} dx = dt$ </div> $\int \frac{1}{(2-t)(2t-1)} dt$ $= \int \frac{1}{-2t^2 + 5t - 2} dt$ $= \frac{-1}{2} \int \frac{1}{t^2 - \frac{5}{2}t + 1} dt$	<p>1/2</p> <p>1/2</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: **22224**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	$= \frac{-1}{2} \int \frac{1}{t^2 - \frac{5}{2}t + \frac{25}{16} - \frac{25}{16} + 1} dt$ $= \frac{-1}{2} \int \frac{1}{\left(t - \frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2} dt$ $= \frac{-1}{2} \frac{1}{2 \cdot \frac{3}{4}} \log \left \frac{t - \frac{5}{4} - \frac{3}{4}}{t - \frac{5}{4} + \frac{3}{4}} \right + c$ $= \frac{-1}{3} \log \left \frac{t - 2}{t - \frac{1}{2}} \right + c$ $= \frac{-1}{3} \log \left \frac{\log x - 2}{\log x - \frac{1}{2}} \right + c$	<p>½</p> <p>1</p> <p>1</p> <p>½</p>
	e)	<p>-----</p> <p>Evaluate: $\int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+4}} dx$</p> <p>Ans $I = \int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+4}} dx$ ----- (1)</p> $I = \int_1^4 \frac{\sqrt[3]{9-(5-x)}}{\sqrt[3]{9-(5-x)} + \sqrt[3]{(5-x)+4}} dx$ $\therefore I = \int_1^4 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4} + \sqrt[3]{9-x}} dx$ ----- (2) <p>add (1) and (2), $I + I = \int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+4}} dx + \int_1^4 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4} + \sqrt[3]{9-x}} dx$</p> $\therefore 2I = \int_1^4 \frac{\sqrt[3]{9-x} + \sqrt[3]{x+4}}{\sqrt[3]{9-x} + \sqrt[3]{x+4}} dx$ $\therefore 2I = \int_1^4 1 dx$ $\therefore 2I = (x)_1^4$ $\therefore I = \frac{3}{2}$	<p>04</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.		Attempt any <u>TWO</u> of the following:	12
	a)	Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{9} + \frac{y^2}{4}$ about x – axis	06
	Ans	Consider $\frac{x^2}{9} + \frac{y^2}{4} = 1$ $\therefore y^2 = \frac{4}{9}(9 - x^2)$ Volume of solid $V = \pi \int_{-a}^a y^2 dx$ $V = \pi \int_{-3}^3 \frac{4}{9}(9 - x^2) dx$ $\therefore V = 2\pi \int_0^3 \frac{4}{9}(9 - x^2) dx$ $\therefore V = \frac{8\pi}{9} \left[9x - \frac{x^3}{3} \right]_0^3$ $\therefore V = \frac{8\pi}{9} \left[\left(9(3) - \frac{3^3}{3} \right) - \left(9(0) - \frac{0^3}{3} \right) \right]$ $V = 16\pi$ (Note :If student has considered/assumed other value than 1 and attempted) (to solve the problem , give appropriate marks.)	1 1 1 1 1 1
	b)	Attempt the following:	06
	(i)	Form the diffrential equation by eliminating the arbitrary constants if $y = a \cos(\log x) + b \sin(\log x)$	03
	Ans	$y = a \cos(\log x) + b \sin(\log x)$ $\therefore \frac{dy}{dx} = -a \sin(\log x) \frac{1}{x} + b \cos(\log x) \frac{1}{x}$ $\therefore x \frac{dy}{dx} = -a \sin(\log x) + b \cos(\log x)$ $\therefore x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = -a \cos(\log x) \frac{1}{x} - b \sin(\log x) \frac{1}{x}$ $\therefore x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -(a \cos(\log x) + b \sin(\log x))$	1 1 ½



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)(i)	$\therefore x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -y$ $\therefore x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$	½
	b)(ii)	Solve the differential equation: $\frac{dy}{dx} + y \tan x = \cos^2 x$	03
	Ans	$\frac{dy}{dx} + y \tan x = \cos^2 x$ <p>Comparing with $\frac{dy}{dx} + Py = Q$</p>	½
		$\therefore P = \tan x \text{ and } Q = \cos^2 x$	½
		$IF = e^{\int \tan x dx} = e^{\log(\sec x)} = \sec x$	½
		$\therefore y \cdot IF = \int Q \cdot IF dx + c$	1
		$y \cdot \sec x = \int \cos^2 x \sec x dx + c$	1
		$y \cdot \sec x = \int \cos x dx + c$	1
		$y \cdot \sec x = \sin x + c$	1
	c)	In a single closed electrical circuit the current 'I' at time t is given by	06
	$E - RI - L \frac{dI}{dt} = 0.$ Find the current I at time t, given that t=0, I=0 and L,R,E are constants.		
Ans	$E - RI - L \frac{dI}{dt} = 0$		
	$\therefore \frac{dI}{dt} + \frac{R}{L} I = \frac{E}{L}$ <p>Comparing with $\frac{dy}{dx} + Py = Q$</p>	½	
	$\therefore P = \frac{R}{L} \text{ and } Q = \frac{E}{L}$		
	$IF = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$	1	
	$\therefore I \cdot IF = \int Q \cdot IF dt + c$		
	$I \cdot e^{\frac{Rt}{L}} = \int \frac{E}{L} e^{\frac{Rt}{L}} dt + c$	1	
	$I \cdot e^{\frac{Rt}{L}} = \frac{E}{L} \frac{e^{\frac{Rt}{L}}}{\frac{R}{L}} + c$	1	
	$I \cdot e^{\frac{Rt}{L}} = \frac{E}{R} e^{\frac{Rt}{L}} + c$		
	When t = 0, I = 0		



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$\therefore c = -\frac{E}{R}$ $I \cdot e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} - \frac{E}{R}$ $I = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t} \right)$	<p>1</p> <p>½</p> <p>1</p>
6.		<p>Attempt any <u>TWO</u> of the following:</p>	12
	a)	<p>Attempt the following:</p>	06
	(i)	<p>Solve the following system of by equations by Jacobi's -Iteration method. (Two iterations)</p> $5x + 2y + z = 12, \quad x + 4y + 2z = 15, \quad x + 2y + 5z = 20$	03
	Ans	$x = \frac{1}{5}(12 - 2y - z)$ $y = \frac{1}{4}(15 - x - 2z)$ $z = \frac{1}{5}(20 - x - 2y)$ <p>Starting with $x_0 = y_0 = z_0 = 0$</p> $x_1 = 2.4$ $y_1 = 3.75$ $z_1 = 4$ $x_2 = 0.1$ $y_2 = 1.15$ $z_2 = 2.02$	<p>1</p> <p>1</p> <p>1</p>
	a(ii)	<p>Solve the following system of equation by using Gauss-Seidel method. (Two iterations)</p> $15x + 2y + z = 18, \quad 2x + 20y - 3z = 19, \quad 3x - 6y + 25z = 22$	03
	Ans	$x = \frac{1}{15}(18 - 2y - z)$ $y = \frac{1}{20}(19 - 2x + 3z)$ $z = \frac{1}{25}(22 - 3x + 6y)$	1



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	Find the approximate root of the equation $x^4 - x - 10 = 0$, by Newton-Raphson method (Carry out four iterations)	
	Ans	Let $f(x) = x^4 - x - 10$ $f(1) = -10 < 0$ $f(2) = 4 > 0$ $f'(x) = 4x^3 - 1$ Initial root $x_0 = 2$ $\therefore f'(2) = 31$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 1.871$ $x_2 = 1.871 - \frac{f(1.871)}{f'(1.871)} = 1.856$ $x_3 = 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856$ $x_4 = 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856$	1 1 1 1
		<i>OR</i>	
		Let $f(x) = x^4 - x - 10$ $f(1) = -10 < 0$ $f(2) = 4 > 0$ $f'(x) = 4x^3 - 1$ Initial root $x_0 = 2$ $\therefore f'(2) = 31$ $x_i = \frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(4x^3 - 1) - [x^4 - x - 10]}{4x^3 - 1}$ $= \frac{3x^4 + 10}{4x^3 - 1}$	1 1 2
		$x_1 = 1.871$ $x_2 = 1.856$ $x_3 = 1.856$ $x_4 = 1.856$	½ ½ ½ ½
		<i>OR</i>	



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	<p>Let $f(x) = x^4 - x - 10$</p> <p>$f(-1) = -8 < 0$</p> <p>$f(-2) = 8 > 0$</p> <p>$f'(x) = 4x^3 - 1$</p> <p>Initial root $x_0 = -2$</p> <p>$\therefore f'(-2) = -33$</p> <p>$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -2 - \frac{f(-2)}{f'(-2)} = -1.758$</p> <p>$x_2 = -1.758 - \frac{f(-1.758)}{f'(-1.758)} = -1.700$</p> <p>$x_3 = -1.700 - \frac{f(-1.700)}{f'(-1.700)} = -1.697$</p> <p>$x_4 = -1.697 - \frac{f(-1.697)}{f'(-1.697)} = -1.697$</p> <p style="text-align: center;">OR</p> <p>Let $f(x) = x^4 - x - 10$</p> <p>$f(-1) = -8 < 0$</p> <p>$f(-2) = 8 > 0$</p> <p>$f'(x) = 4x^3 - 1$</p> <p>Initial root $x_0 = -2$</p> <p>$\therefore f'(-2) = -33$</p> <p>$x_i = \frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(4x^3 - 1) - [x^4 - x - 10]}{4x^3 - 1}$</p> <p>$= \frac{3x^4 + 10}{4x^3 - 1}$</p> <p>$x_1 = -1.758$</p> <p>$x_2 = -1.700$</p> <p>$x_3 = -1.697$</p> <p>$x_4 = -1.697$</p> <p>-----</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
		<p><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p> <p>-----</p> <p>-----</p>	