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WINTER- 19 EXAMINATION

Subject Name: Applied Mathematics

Model Answer

Subject Code:

22224

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$ if $y = x^2 \cdot e^x$	02
	Ans	$y = x^2 \cdot e^x$ $\frac{dy}{dx} = x^2 \cdot e^x + e^x \cdot 2x$ $\frac{dy}{dx} = xe^x(x+2)$	1 1
	d)	Evaluate $\int [e^x + a^x + x^a + a^a] dx$	02
	Ans	$\int [e^x + a^x + x^a + a^a] dx$ $= e^x + \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$	2
e)	Evaluate: $\int \left[\frac{1}{1 + \cos 2x} \right] dx$	02	
Ans	$\int \left[\frac{1}{1 + \cos 2x} \right] dx$ $= \int \left[\frac{1}{2 \cos^2 x} \right] dx$ $= \frac{1}{2} \int \sec^2 x dx$ $= \frac{1}{2} \tan x + c$	1 1	
f)	Find the area bounded by $y = x$, X-axis and $x = 0$ to $x = 4$.	02	
Ans	Area $A = \int_a^b y dx$ $= \int_0^4 x dx$ $= \left[\frac{x^2}{2} \right]_0^4$	½ ½	



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1.		$= \left(\frac{4^2}{2} - 0 \right)$ $= 8$	$\frac{1}{2}$ $\frac{1}{2}$
	g)	Find a real root of the equation $x^3 + 4x - 9 = 0$ in the interval (1, 2) by using Bisection method. (only one iteration)	02
	Ans	Let $f(x) = x^3 + 4x - 9$ $f(1) = -4$ $f(2) = 7$ \therefore the root is in (1, 2) $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$	1 1
2		Solve any <u>THREE</u> of the following:	12
	a)	Find $\frac{dy}{dx}$, if $y = \frac{5e^x}{3e^x + 1}$ at $x = 0$	04
	Ans	$y = \frac{5e^x}{3e^x + 1}$ $\frac{dy}{dx} = \frac{(3e^x + 1)5e^x - 5e^x(3e^x)}{(3e^x + 1)^2}$ $\frac{dy}{dx} = \frac{15e^{2x} + 5e^x - 15e^{2x}}{(3e^x + 1)^2}$ $\frac{dy}{dx} = \frac{5e^x}{(3e^x + 1)^2}$ at $x = 0$ $\frac{dy}{dx} = \frac{5e^0}{(3e^0 + 1)^2}$ $= \frac{5}{16} \text{ or } 0.3125$	2 1 1



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2.	b)	If $x = a(1 + \cos \theta)$, $y = a(1 - \cos \theta)$, find $\frac{dy}{dx}$	04
	Ans	$x = a(1 + \cos \theta)$, $y = a(1 - \cos \theta)$ $\frac{dx}{d\theta} = a(-\sin \theta) = -a \sin \theta$, $\frac{dy}{d\theta} = a(0 + \sin \theta) = a \sin \theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{-a \sin \theta}$ $\frac{dy}{dx} = -1$	1+1 1 1
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	04
	Ans	Let length of rectangle = x , breadth = y $\therefore 2x + 2y = 36$ $\therefore y = 18 - x$ Area $A = x \times y$ $A = x(18 - x)$ $\therefore A = 18x - x^2$ $\therefore \frac{dA}{dx} = 18 - 2x$ $\therefore \frac{d^2A}{dx^2} = -2$ Let $\frac{dA}{dx} = 0$ $\therefore 18 - 2x = 0$ $\therefore x = 9$ at $x = 9$ $\frac{d^2A}{dx^2} = -2 < 0$ Area is maximum at $x = 9$ Length = 9 ; breadth = 9	1 1 1/2 1/2 1/2 1/2



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3.	a)	$\therefore \frac{dy}{dx} = \frac{-2}{9}$ $\therefore \text{slope of tangent, } m = \frac{-2}{9}$ <p>Equation of tangent at (1,2) is</p> $y - 2 = \frac{-2}{9}(x - 1)$ $\therefore 9y - 18 = -2x + 2$ $\therefore 2x + 9y - 20 = 0$ $\therefore \text{slope of normal, } m' = \frac{-1}{m} = \frac{9}{2}$ <p>Equation of normal at (1,2) is</p> $y - 2 = \frac{9}{2}(x - 1)$ $\therefore 2y - 4 = 9x - 9$ $\therefore 9x - 2y - 5 = 0$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
	b)	<p>Find $\frac{dy}{dx}$ if $y = \tan^{-1} \left[\frac{2x}{1+35x^2} \right]$</p>	04
	Ans	$y = \tan^{-1} \left[\frac{7x - 5x}{1 + 7x \cdot 5x} \right]$ $y = \tan^{-1} 7x - \tan^{-1} 5x$ $\frac{dy}{dx} = \frac{7}{1+49x^2} - \frac{5}{1+25x^2}$	<p>1</p> <p>1</p> <p>2</p>
c)	<p>If $x^y = e^{x-y}$ Show that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$</p>	04	
Ans	$x^y = e^{x-y}$ $\log x^y = \log e^{x-y}$ $y \log x = x - y \log e$ $y \log x = x - y$ $y \log x + y = x$	<p>1/2</p> <p>1/2</p> <p>1/2</p>	



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3.	c)	$y(\log x + 1) = x$ $y = \frac{x}{\log x + 1}$ $\frac{dy}{dx} = \frac{(\log x + 1) \cdot 1 - x \cdot \frac{1}{x}}{(\log x + 1)^2}$ $= \frac{\log x}{(\log x + 1)^2}$	1 1 ½
	d)	<p>Evaluate $\int \frac{dx}{5 + 3 \cos 2x}$</p> <p>Ans $\int \frac{dx}{5 + 3 \cos 2x}$</p> <p>Put $\tan x = t$, $dx = \frac{dt}{1+t^2}$</p> $\cos 2x = \frac{1-t^2}{1+t^2}$ $\int \frac{\frac{dt}{1+t^2}}{5 + 3 \left(\frac{1-t^2}{1+t^2} \right)}$ $= \int \frac{dt}{5(1+t^2) + 3(1-t^2)}$ $= \int \frac{dt}{5 + 5t^2 + 3 - 3t^2}$ $= \int \frac{dt}{2t^2 + 8}$ $= \int \frac{dt}{(\sqrt{2}t)^2 + (\sqrt{8})^2} \quad \text{OR} \quad = \frac{1}{2} \int \frac{dt}{t^2 + 4}$ $= \frac{1}{\sqrt{8}} \tan^{-1} \left(\frac{\sqrt{2}t}{\sqrt{8}} \right) \cdot \frac{1}{\sqrt{2}} + c \quad = \frac{1}{2} \cdot \frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) + c$	04 1 1 1



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3.	d)	$= \frac{1}{\sqrt{16}} \tan^{-1} \left(\frac{\sqrt{2} \tan x}{\sqrt{8}} \right) + c \quad \text{OR} \quad = \frac{1}{4} \tan^{-1} \left(\frac{t}{2} \right) + c$ $= \frac{1}{4} \tan^{-1} \left(\frac{\tan x}{2} \right) + c \quad \text{OR} \quad = \frac{1}{4} \tan^{-1} \left(\frac{\tan x}{2} \right) + c$	<p>½</p> <p>½</p>
4.		<p>Solve any THREE of the following:</p>	12
	a)	<p>Evaluate $\int \frac{[e^x(x+1)]}{\cos^2(x.e^x)} dx$</p>	04
	Ans	$\int \frac{[e^x(x+1)]}{\cos^2(x.e^x)} dx$ <p>Put $x.e^x = t$ $\therefore (x.e^x + e^x.1)dx = dt$ $[e^x(x+1)]dx = dt$ $\therefore \int \frac{1}{\cos^2 t} dt$ $= \int \sec^2 t dt$ $= \tan t + c$ $= \tan(x.e^x) + c$</p>	<p>1</p> <p>1</p> <p>1</p> <p>½</p> <p>½</p>
	b)	<p>Evaluate: $\int \frac{dx}{2x^2 + 3x + 2}$</p>	04
	Ans	$\int \frac{dx}{2x^2 + 3x + 2}$ $= \frac{1}{2} \int \frac{dx}{x^2 + \frac{3}{2}x + 1}$ $= \frac{1}{2} \int \frac{dx}{x^2 + \frac{3}{2}x + \frac{9}{16} + 1 - \frac{9}{16}}$	<p>½</p> <p>1</p>



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4.	b)	$= \frac{1}{2} \int \frac{dx}{\left(x + \frac{3}{4}\right)^2 + \frac{7}{16}}$ $= \frac{1}{2} \int \frac{dx}{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$ $= \frac{1}{2} \frac{1}{\frac{\sqrt{7}}{4}} \tan^{-1} \left(\frac{x + \frac{3}{4}}{\frac{\sqrt{7}}{4}} \right) + c$ $= \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{4x + 3}{\sqrt{7}} \right) + c$	<p>½</p> <p>1</p> <p>1</p>
	c)	<p>-----</p> <p>Evaluate $\int x^2 \cdot \tan x \, dx$</p>	04
	Ans	$\int x^2 \cdot \tan x \, dx$ $= x^2 \left(\int \tan x \, dx \right) - \int \left(\int \tan x \, dx \cdot \frac{d}{dx}(x^2) \right) dx$ $= x^2 \log(\sec x) - \int \log(\sec x) \cdot 2x \, dx$ $= x^2 \log(\sec x) - 2 \left[\log(\sec x) \frac{x^2}{2} - \int \frac{1}{\sec x} \cdot \sec x \tan x \cdot \frac{x^2}{2} dx \right]$ $= x^2 \log(\sec x) - 2 \left[\log(\sec x) \frac{x^2}{2} - \frac{1}{2} \int x^2 \cdot \tan x \, dx \right]$ $= x^2 \log(\sec x) - 2 \left[\log(\sec x) \frac{x^2}{2} - \frac{1}{2} I \right]$ $I = x^2 \log(\sec x) - \log(\sec x) x^2 + I$	<p>½</p> <p>1</p> <p>½</p> <p>½</p>
		<p>Note: If students attempted to solve the question give appropriate marks.</p> <p>-----</p>	
	d)	<p>Evaluate $\int \frac{\sec^2 x}{(\tan x)(\tan x + 1)} dx$</p>	04



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4.	Ans	$\int \frac{\sec^2 x}{(\tan x)(\tan x + 1)} dx$ <p>Put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $\therefore \int \frac{1}{(t)(t+1)} dt$ $\frac{1}{(t)(t+1)} = \frac{A}{t} + \frac{B}{t+1}$ $\therefore 1 = A(t+1) + B(t)$ \therefore Put $t = 0$, $A = 1$ Put $t = -1$, $B = -1$ $\therefore \frac{1}{(t)(t+1)} = \frac{1}{t} - \frac{1}{t+1}$ $\therefore \int \frac{1}{(t)(t+1)} dt = \int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt$ $= \log(t) - \log(t+1) + c$ $= \log(\tan x) - \log(\tan x + 1) + c$</p>	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>
	e) Ans	<p>Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx$</p> $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx$ $= \int_0^{\frac{\pi}{2}} \frac{1}{1 + \frac{\sqrt{\sin x}}{\sqrt{\cos x}}} dx$ <p>Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \dots \dots \dots (1)$</p>	<p>04</p>



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Subject Code: 22224

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4.	e)	$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)} + \sqrt{\sin\left(\frac{\pi}{2}-x\right)}} dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \dots\dots\dots (2)$ <p>Add (1) and (2)</p> $I+I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ $2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ $2I = \int_0^{\frac{\pi}{2}} dx$ $2I = [x]_0^{\frac{\pi}{2}}$ $2I = \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$	<p>1</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>
5	a)	<p>Solve any TWO of the following:</p> <p>Find area bounded by the curve $y = x^2$ and the line $y = x$</p> <p>We have $y = x^2$ and $y = x$</p> $\therefore x^2 - x = 0$ $\therefore x(x-1) = 0$ $\therefore x = 0 \text{ or } x = 1$ $\text{Area} = \int_a^b (y_1 - y_2) dx$ $= \int_0^1 (x^2 - x) dx$	<p>12</p> <p>06</p> <p>1</p> <p>1</p>



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5.	a)	$= \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_0^1$ $= \left[\frac{1^3}{3} - \frac{1^2}{2} - 0 \right]$ $= -\frac{1}{6}$ <p>$\therefore A = \frac{1}{6}$ or 0.167 (\because Area is always +ve)</p>	1 1 1 1
	b)	Attempt the following:	06
	i)	From the differential equation by eliminating the arbitrary constant if	03
	Ans	$y = A \cos x + B \sin x.$ $y = A \cos x + B \sin x.$ $\frac{dy}{dx} = -A \sin x + B \cos x$ $\frac{d^2y}{dx^2} = -A \cos x - B \sin x$ $= -(A \cos x + B \sin x)$ $= -y$ $\frac{d^2y}{dx^2} + y = 0$	1 1 1
	ii)	Solve $(1+x^2)dy - x^2.ydx = 0$	03
	Ans	$(1+x^2)dy - x^2.ydx = 0$ $(1+x^2)dy = x^2.ydx$ $\frac{dy}{y} = \frac{x^2 dx}{1+x^2}$ $\int \frac{dy}{y} = \int \frac{x^2 dx}{1+x^2}$ $\int \frac{dy}{y} = \int \frac{1+x^2-1 dx}{1+x^2}$	1



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5.		$\int \frac{dy}{y} = \int \left[1 - \frac{1}{1+x^2} \right] dx$ $\log y = x - \tan^{-1} x + c$	1 1
	c) Ans	<p>Solve the D.E $\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R}$ given that $q = 0$ when $t = 0$ and E,R,C are constant</p> $\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R}$ $I.F = e^{\int \frac{1}{RC} dt}$ $= e^{\frac{t}{RC}}$ $\therefore q.e^{\frac{t}{RC}} = \int \frac{E}{R}.e^{\frac{t}{RC}} dt$ $= \frac{E}{R} e^{\frac{t}{RC}} \cdot \frac{1}{\frac{1}{RC}} + c_1$ $q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}} EC + c_1$ <p>given that $q = 0$ when $t = 0$</p> $0 = e^0 EC + c_1$ $c_1 = -EC$ $q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}} EC - EC$ $q = EC \left(1 - e^{-\frac{t}{RC}} \right)$	06 1 1 1 1 1
6.		Solve any TWO of the following:	12
	a) i)	<p>Attempt the following:</p> <p>Solve the equations by Gauss-Seidal method. (two iterations only)</p> $10x + y + 2z = 13, \quad 3x + 10y + z = 14, \quad 2x + 3y + 10z = 15$	06 03



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6.	c)	<p>Initial root $x_0=2$ $\therefore f'(2)=5$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 1.8$ $x_2 = 1.8 - \frac{f(1.8)}{f'(1.8)} = 1.7913$ $x_3 = 1.7913 - \frac{f(1.7913)}{f'(1.7913)} = 1.7912$ $x_4 = 1.7912 - \frac{f(1.7912)}{f'(1.7912)} = 1.7912$ OR Let $f(x) = x^2 + x - 5$ $f(1) = -3 < 0$ $f(2) = 1 > 0$ $f'(x) = 2x + 1$ Initial root $x_0=2$ $x_i = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 + x - 5}{2x + 1}$ $= \frac{2x^2 + x - x^2 - x + 5}{2x + 1}$ $= \frac{x^2 + 5}{2x + 1}$ $x_1 = 1.8$ $x_2 = 1.7913$ $x_3 = 1.7912$ $x_4 = 1.7912$</p>	<p>1 1 1 1 1 1 2 ½ ½ ½ ½</p>
		<p><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>	